

NMSU MATH PROBLEM OF THE WEEK

Solution to Problem 5

Fall 2023

Problem 5

Compute

$$\frac{1}{1} + \frac{1}{1+2} + \frac{1}{1+2+3} + \cdots + \frac{1}{1+2+\cdots+100}.$$

Solution. Note that $1+2+\cdots+k = \frac{k(k+1)}{2}$ for any natural number k . Therefore, we may write

$$\frac{1}{1+\cdots+k} = \frac{2}{k(k+1)} = 2\left(\frac{1}{k+1} - \frac{1}{k}\right).$$

Thus

$$\begin{aligned} \frac{1}{1} + \frac{1}{1+2} + \cdots + \frac{1}{1+\cdots+100} &= \sum_{k=1}^{100} 2\left(\frac{1}{k} - \frac{1}{k+1}\right) \\ &= 2\left(\left(\frac{1}{1} - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \cdots + \left(\frac{1}{99} - \frac{1}{100}\right) + \left(\frac{1}{100} - \frac{1}{101}\right)\right) \\ &= 2\left(1 - \frac{1}{101}\right) \\ &= \frac{200}{101}. \end{aligned}$$

■